

EGT through Quantum Mechanics & from Statistical Physics to Economics

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By analyzing the relationships between a socioeconomical system modeled through evolutionary game theory and a physical system modeled through quantum mechanics we show how although both systems are described through two theories apparently different both are analogous and thus exactly equivalents. The extensions of quantum mechanics to statistical physics and information theory let us use some of their definitions for the best understanding of the behavior of economics and biology. The quantum analogue of the replicator dynamics is the von Neumann equation. A system in where all its members are in Nash equilibrium is equivalent to a system in a maximum entropy state. Nature is a game in where its players compete for a common welfare and the equilibrium of the system that they are members. They act as a whole besides individuals like they obey a rule in where they prefer to work for the welfare of the collective besides the individual welfare.

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I. INTRODUCTION

Why has it been possible to apply some methods of statistical physics to economics and biology? It is a good reason to say that physics is a model which tries to describe phenomena and behaviors and if this model fits and describes almost perfectly the observed and the measured even in the economic world then there is no problem or impediment to apply physics to solve problems in economics and biology. But, could economics, biology and statistical physics be correlated? Could it have a relationship between quantum mechanics and game theory? or could quantum mechanics even enclose theories like games and the evolutionary dynamics? This possibility could make quantum mechanics a theory more general that we have thought.

Problems in economy and finance have attracted the interest of statistical physicists. Ausloos et al [1] analyzed fundamental problems pertain to the existence or not long-, medium-, short-range power-law correlations in economic systems as well as to the presence of financial cycles. They recalled methods like the extended detrended fluctuation analysis and the multi-affine analysis emphasizing their value in sorting out correlation ranges and predictability. They also indicated the possibility of crash predictions. They showed the well known financial analyst technique, the so called moving average, to raise questions about fractional Brownian motion properties. The (m, k) -Zipf method and the i -variability diagram technique were presented for sorting out short range correlations.

J.-P. Bouchaud [2] analyzed three main themes in the field of statistical finance, also called econophysics: (i) empirical studies and the discovery of universal features in the statistical texture of financial time series, (ii) the use of these empirical results to devise better models of

risk and derivative pricing, of direct interest for the financial industry, and (iii) the study of “agent-based models” in order to unveil the basic mechanisms that are responsible for the statistical “anomalies” observed in financial time series.

Statistical physicists are also extremely interested in fluctuations [3]. One reason physicists might want to quantify economic fluctuations is in order to help our world financial system avoid “economic earthquakes”. Also it is suggested that in the field of turbulence, we may find some crossover with certain aspects of financial markets.

Kobелеv et al [4] used methods of statistical physics of open systems for describing the time dependence of economic characteristics (income, profit, cost, supply, currency, etc.) and their correlations with each other. They also offered nonlinear equations (analogies of known reaction-diffusion, kinetic, Langevin equation) to describe appearance of bifurcations, self-sustained oscillational processes, self-organizations in economic phenomena.

It is generally accepted that entropy can be used for the study of economic systems consisting of large number of components [5]. I. Antoniou et al [6] introduced a new approach for the presentation of economic systems with a small number of components as a statistical system described by density functions and entropy. This analysis is based on a Lorenz diagram and its interpolation by a continuous function. Conservation of entropy in time may indicate the absence of macroscopic changes in redistribution of resources. Assuming the absence of macro-changes in economic systems and in related additional expenses of resources, we may consider the entropy as an indicator of efficiency of the resources distribution.

This approach is not limited by the number of components of the economic system and can be applied to wide class of economic problems. They think that the bridge between distribution of resources and proposed probability distributions may permit us to use the methods of nonequilibrium statistical mechanics for the study and forecast of the dynamics of complex economic systems and to make correct management decisions.

Statistical mechanics and economics study big ensembles: collections of atoms or economic agents, respectively. The fundamental law of equilibrium statistical mechanics is the Boltzmann-Gibbs law, which states that the probability distribution of energy E is $P(E) = Ce^{-E/T}$, where T is the temperature, and C is a normalizing constant. The main ingredient that is essential for the derivation of the Boltzmann-Gibbs law is the conservation of energy. Thus, one may generalize that any conserved quantity in a big statistical system should have an exponential probability distribution in equilibrium [7]. In a closed economic system, money is conserved. Thus, by analogy with energy, the equilibrium probability distribution of money must follow the exponential Boltzmann-Gibbs law characterized by an effective temperature equal to the average amount of money per economic agent. Drăgulescu and Yakovenko demonstrated how the Boltzmann-Gibbs distribution emerges in computer simulations of economic models. They considered a thermal machine, in which the difference of temperature allows one to extract a monetary profit. They also discussed the role of debt, and models with broken time-reversal symmetry for which the Boltzmann-Gibbs law does not hold.

Recently the insurance market, which is one of the important branches of economy, have attracted the attention of physicists [8]. Some concepts of the statistical mechanics, specially the maximum entropy principle is used for pricing the insurance. Darooneh obtained the price density based on this principle, applied it to multi agents model of insurance market and derived the utility function. The main assumption in his work is the correspondence between the concept of the equilibrium in physics and economics. He proved that economic equilibrium can be viewed as an asymptotic approximation to physical equilibrium and some difficulties with mechanical picture of the equilibrium may be improved by considering the statistical description of it. Topsøe [9] also has suggested that thermodynamical equilibrium equals game theoretical equilibrium.

In this paper we try to find a deeper relationship between quantum mechanics and game theory. By using some elements of evolution that are contained inside evolutionary game theory through the replicator dynamics, game theory is generalized. Evolutionary game theory is compared with quantum mechanics and it is shown how although both systems analyzed are described through

two theories apparently different both are analogous and thus exactly equivalents. So, we could make use of some of their concepts, laws and definitions for the best understanding of the behavior of economics and biology. The quantum analogue of the replicator dynamics is the von Neumann equation. A system in where all its members are in Nash equilibrium is equivalent to a system in a maximum entropy state. Nature is a game in where its players act as a whole besides individuals like they obey a rule in where they prefer to work for the welfare of the collective besides the individual welfare. Should a socioeconomical system also satisfy a Collective Welfare Principle?

II. CLASSICAL, EVOLUTIONARY & QUANTUM GAMES

Game theory [10, 11, 12] is the study of decision making of competing agents in some conflict situation. It tries to understand the birth and the development of conflicting or cooperative behaviors among a group of individuals who behave rationally and strategically according to their personal interests. Each member in the group strive to maximize its welfare by choosing the best courses of strategies from a cooperative or individual point of view.

The central equilibrium concept in game theory is the Nash Equilibrium. A *Nash equilibrium* (NE) is a set of strategies, one for each player, such that no player has an incentive to unilaterally change his action. Players are in equilibrium if a change in strategies by any one of them would lead that player to earn less than if he remained with his current strategy. A *Nash equilibrium* satisfies the following condition

$$E(p, p) \geq E(r, p), \quad (1)$$

where $E(s_i, s_j)$ is a real number that represents the payoff obtained by a player who plays the strategy s_i against an opponent who plays the strategy s_j . A player can not increase his payoff if he decides to play the strategy r instead of p .

Evolutionary game theory [13, 14, 15] does not rely on rational assumptions but on the idea that the Darwinian process of natural selection [16] drives organisms towards the optimization of reproductive success [17]. Instead of working out the optimal strategy, the different phenotypes in a population are associated with the basic strategies that are shaped by trial and error by a process of natural selection or learning. The natural selection process that determines how populations playing specific strategies evolve is known as the replicator dynamics [14, 15, 18, 19] whose stable fixed points are Nash equilibria [11]. The central equilibrium concept of evolutionary game theory is the notion of Evolutionary Stable Strategy introduced by J. Smith and G. Price [13, 20].

An ESS is described as a strategy which has the property that if all the members of a population adopt it, no mutant strategy could invade the population under the influence of natural selection. ESS are interpreted as stable results of processes of natural selection.

Consider a large population in which a two person game $G = (S, E)$ is played by randomly matched pairs of animals generation after generation. Let p be the strategy played by the vast majority of the population, and let r be the strategy of a mutant present in small frequency. Both p and r can be pure or mixed. An *evolutionary stable strategy* (ESS) p of a symmetric two-person game $G = (S, E)$ is a pure or mixed strategy for G which satisfies the following two conditions

$$\begin{aligned} E(p, p) &> E(r, p), \\ \text{If } E(p, p) &= E(r, p) \text{ then } E(p, r) > E(r, r). \end{aligned} \quad (2)$$

Since the stability condition only concerns to alternative best replies, p is always evolutionarily stable if (p, p) is a strict equilibrium point. An ESS is also a Nash equilibrium since is the best reply to itself and the game is symmetric. The set of all the strategies that are ESS is a subset of the NE of the game. A population which plays an ESS can withstand an invasion by a small group of mutants playing a different strategy. It means that if a few individuals which play a different strategy are introduced into a population in an ESS, the evolutionarily selection process would eventually eliminate the invaders.

Quantum games have proposed a new point of view for the solution of the classical problems and dilemmas in game theory. Quantum games are more efficient than classical games and provide a saturated upper bound for this efficiency [21, 22, 23, 24, 25, 26].

III. REPLICATOR DYNAMICS & EGT

The model used in EGT is the following: Each agent in a n -player game where the i^{th} player has as strategy space S_i is modeled by a population of players which have to be partitioned into groups. Individuals in the same group would all play the same strategy. Randomly we make play the members of the subpopulations against each other. The subpopulations that perform the best will grow and those that do not will shrink and eventually will vanish. The process of natural selection assures survival of the best players at the expense of the others. The natural selection process that determines how populations playing specific strategies evolve is known as the replicator dynamics

$$\frac{dx_i}{dt} = [f_i(x) - \langle f(x) \rangle] x_i, \quad (3)$$

$$\frac{dx_i}{dt} = \left[\sum_{j=1}^n a_{ij} x_j - \sum_{k,l=1}^n a_{kl} x_k x_l \right] x_i. \quad (4)$$

The probability of playing certain strategy or the relative frequency of individuals using that strategy is denoted by frequency x_i . The fitness function $f_i = \sum_{j=1}^n a_{ij} x_j$ specifies how successful each subpopulation is, $\langle f(x) \rangle = \sum_{k,l=1}^n a_{kl} x_k x_l$ is the average fitness of the population, and a_{ij} are the elements of the payoff matrix A . The replicator dynamics rewards strategies that outperform the average by increasing their frequency, and penalizes poorly performing strategies by decreasing their frequency. The stable fixed points of the replicator dynamics are Nash equilibria, it means that if a population reaches a state which is a Nash equilibrium, it will remain there.

We can represent the replicator dynamics in matrix form

$$\frac{dX}{dt} = G + G^T. \quad (5)$$

The relative frequencies matrix X has as elements

$$x_{ij} = (x_i x_j)^{1/2} \quad (6)$$

and

$$\begin{aligned} (G + G^T)_{ij} &= \frac{1}{2} \sum_{k=1}^n a_{ik} x_k x_{ij} \\ &+ \frac{1}{2} \sum_{k=1}^n a_{jk} x_k x_{ji} \\ &- \sum_{k,l=1}^n a_{kl} x_k x_l x_{ij} \end{aligned} \quad (7)$$

are the elements of the matrix $(G + G^T)$.

From this matrix representation we can find a Lax representation of the replicator dynamics [27]

$$\frac{dX}{dt} = [[Q, X], X], \quad (8)$$

$$\frac{dX}{dt} = [\Lambda, X]. \quad (9)$$

The matrix Λ is equal to $\Lambda = [Q, X]$ with

$$(\Lambda)_{ij} = \frac{1}{2} \left[\left(\sum_{k=1}^n a_{ik} x_k \right) x_{ij} - x_{ji} \left(\sum_{k=1}^n a_{jk} x_k \right) \right] \quad (10)$$

and Q is a diagonal matrix which has as elements

$$q_{ii} = \frac{1}{2} \sum_{k=1}^n a_{ik} x_k. \quad (11)$$

IV. RELATIONSHIPS BETWEEN QUANTUM MECHANICS & GAME THEORY

In table 1 we compare some characteristic aspects of quantum mechanics and game theory.

Table 1

Quantum Mechanics	Game Theory
n system members	n players
Quantum states	Strategies
Density operator	Relative frequencies vector
Von Neumann equation	Replicator Dynamics
Von Neumann entropy	Shannon entropy
System Equilibrium	Payoff
Maximum entropy	Maximum payoff
“Altruism”	Altruism or selfish
Collective Welfare principle	Minority Welfare principle

It is easy to realize the clear resemblances and apparent differences between both theories and between the properties both enjoy. This was a motivation to try to find an actual relationship between both systems.

We have to remember that Schrödinger equation describes only the evolution of pure states in quantum mechanics. To describe correctly a statistical mixture of states it is necessary the introduction of the density operator

$$\rho(t) = \sum_{i=1}^n p_i |\Psi_i(t)\rangle \langle \Psi_i(t)| \quad (12)$$

which contains all the information of the statistical system. The time evolution of the density operator is given by the von Neumann equation

$$i\hbar \frac{d\rho}{dt} = [\hat{H}, \rho] \quad (13)$$

which is only a generalization of the Schrödinger equation and the quantum analogue of Liouville’s theorem.

Evolutionary game theory has been applied to the solution of games from a different perspective. Through the replicator dynamics it is possible to solve not only evolutionary but also classical games. That is why EGT has been considered like a generalization of classical game theory. The cornerstone of EGT is the concept of evolutionary stable strategy (ESS) that is a strengthened notion of Nash equilibrium. The evolution of relative frequencies in a population is given by the replicator dynamics. In a recent work we showed that vectorial equation can be represented in a matrix commutative form (9). This matrix commutative form follows the same dynamic than the von Neumann equation and the properties of its correspondent elements (matrixes) are similar, being the properties corresponding to our quantum system more general than the classical system.

The next table shows some specific resemblances between quantum statistical mechanics and evolutionary game theory.

Table 2

Quantum Statistical Mechanics	Evolutionary Game Theory
n system members	n population members
Each member in the state $ \Psi_k\rangle$	Each member plays strategy s_i
$ \Psi_k\rangle$ with $p_k \rightarrow \rho_{ii}$	$s_i \rightarrow x_i$
$\rho, \sum_i \rho_{ii} = 1$	$X, \sum_i x_i = 1$
$i\hbar \frac{d\rho}{dt} = [\hat{H}, \rho]$	$\frac{dX}{dt} = [\Lambda, X]$
$S = -Tr\{\rho \ln \rho\}$	$H = -\sum_i x_i \ln x_i$

Both systems are composed by n members (particles, subsystems, players, states, etc.). Each member is described by a state or a strategy which has assigned a determined probability. The quantum mechanical system is described by the density operator ρ whose elements represent the system average probability of being in a determined state. For evolutionary game theory, we defined a relative frequencies matrix X to describe the system whose elements can represent the frequency of players playing a determined strategy. The evolution of the density operator is described by the von Neumann equation which is a generalization of the Schrödinger equation. While the evolution of the relative frequencies in a population is described by the Lax form of the replicator dynamics which is a generalization of the replicator dynamics in vectorial form.

In table 3 we show the properties of the matrixes ρ and X .

Table 3

Density Operator	Relative freq. Matrix
ρ is Hermitian	X is Hermitian
$Tr\rho(t) = 1$	$TrX = 1$
$\rho^2(t) \leq \rho(t)$	$X^2 = X$
$Tr\rho^2(t) \leq 1$	$TrX^2(t) = 1$

Although both systems analyzed are described through two apparently different theories (quantum mechanics and game theory), both systems are analogous and thus exactly equivalents [28, 29]. So, we can take some concepts, laws and definitions from quantum mechanics and physics for the best understanding of the behavior of economics and biology. Quantum mechanics could be used to explain more correctly biological and economical processes and even it could enclose theories like games and evolutionary dynamics [27, 28, 29]. This could make quantum mechanics a more general theory that we had thought. The resemblances between both systems and the similarity in the properties of their corresponding elements let us to define and propose the next quantization relationships.

V. QUANTUM REPLICATOR DYNAMICS & QUANTIZATION RELATIONSHIPS

Let us propose the next quantization relationships

$$\begin{aligned} x_i &\rightarrow \sum_{k=1}^n \langle i | \Psi_k \rangle p_k \langle \Psi_k | i \rangle = \rho_{ii}, \\ (x_i x_j)^{1/2} &\rightarrow \sum_{k=1}^n \langle i | \Psi_k \rangle p_k \langle \Psi_k | j \rangle = \rho_{ij}. \end{aligned} \quad (14)$$

A population will be represented by a quantum system in which each subpopulation playing strategy s_i will be represented by a pure ensemble in the state $|\Psi_k(t)\rangle$ and with probability p_k . The probability x_i of playing strategy s_i or the relative frequency of the individuals using strategy s_i in that population will be represented as the probability ρ_{ii} of finding each pure ensemble in the state $|i\rangle$ [27].

Through these quantization relationships the replicator dynamics (in matrix commutative form) takes the form of the equation of evolution of mixed states. And also

$$\begin{aligned} X &\longrightarrow \rho, \\ \Lambda &\longrightarrow -\frac{i}{\hbar} \hat{H}, \end{aligned} \quad (15)$$

where \hat{H} is the Hamiltonian of the physical system.

The equation of evolution of mixed states from quantum statistical mechanics (13) is the quantum analogue of the replicator dynamics in matrix commutative form (9) and both systems and their respective matrixes have similar properties. Through these relationships we could describe classical, evolutionary and quantum games and also the biological systems that were described before through evolutionary dynamics with the replicator dynamics.

VI. GAMES FROM QIT

Lets consider a system composed by N members, players, strategies, states, etc. This system is described completely through certain density operator ρ (12), its evolution equation (the von Neumann equation) (13) and its entropy. Classically, the system is described through the matrix of relative frequencies X , the replicator dynamics and the Shannon entropy. For the quantum case we define the von Neumann entropy as [30, 31, 32, 33]

$$S = -Tr \{ \rho \ln \rho \}, \quad (17)$$

and for the classical case

$$H = - \sum_{i=1} x_{ii} \ln x_{ii} \quad (18)$$

which is the Shannon entropy over the relative frequencies vector x (the diagonal elements of X).

The time evolution equation of H assuming that x evolves following the replicator dynamics is [28, 29, 31]

$$\frac{dH(t)}{dt} = Tr \left\{ U(\tilde{H} - X) \right\}. \quad (19)$$

\tilde{H} is a diagonal matrix whose trace is equal to the Shannon entropy and its elements are $\tilde{h}_{ii} = -x_i \ln x_i$. The matrix U has as elements $U_i = \sum_{j=1}^n a_{ij} x_j - \sum_{k,l=1}^n a_{kl} x_l x_k$.

With the purpose of calculating the time evolution of the von Neumann entropy we approximate the logarithm of ρ by series [28, 29, 31] $\ln \rho = (\rho - I) - \frac{1}{2}(\rho - I)^2 + \frac{1}{3}(\rho - I)^3 \dots$ and

$$\begin{aligned} \frac{dS(t)}{dt} &= \frac{11}{6} \sum_i \frac{d\rho_{ii}}{dt} \\ &\quad - 6 \sum_{i,j} \rho_{ij} \frac{d\rho_{ji}}{dt} \\ &\quad + \frac{9}{2} \sum_{i,j,k} \rho_{ij} \rho_{jk} \frac{d\rho_{ki}}{dt} \\ &\quad - \frac{4}{3} \sum_{i,j,k,l} \rho_{ij} \rho_{jk} \rho_{kl} \frac{d\rho_{li}}{dt} + \zeta. \end{aligned} \quad (20)$$

Entropy is the central concept of information theories. The von Neumann entropy [32, 33] is the quantum analogue of Shannon's entropy but it appeared 21 years before and generalizes Boltzmann's expression. Entropy in quantum information theory plays prominent roles in many contexts, e.g., in studies of the classical capacity of a quantum channel [34, 35] and the compressibility of a quantum source [36, 37]. Quantum information theory appears to be the basis for a proper understanding of the emerging fields of quantum computation [38, 39], quantum communication [40, 41], and quantum cryptography [42, 43].

In classical physics, information processing and communication is best described by Shannon information theory. The Shannon entropy expresses the average information we expect to gain on performing a probabilistic experiment of a random variable which takes the values s_i with the respective probabilities x_i . It also can be seen as a measure of uncertainty before we learn the value of that random variable. The Shannon entropy of the probability distribution associated with the source gives the minimal number of bits that are needed in order to store the information produced by a source, in the sense that the produced string can later be recovered.

We can define an entropy over a random variable S^A (player's A strategic space) which can take the values $\{s_i^A\}$ (or $\{|\Psi_i\rangle\}_A$) with the respective probabilities $(x_i)_A$

(or $(\rho_{ij})_A$) [31]. We could interpret the entropy of our game as a measure of uncertainty before we learn what strategy player A is going to use. If we do not know what strategy a player is going to use every strategy becomes equally probable and our uncertainty becomes maximum and greater while greater is the number of strategies. If we would know the relative frequency with player A uses any strategy we can prepare our reply in function of that most probable player A strategy. Obviously our uncertainty vanish if we are sure about the strategy our opponent is going to use.

If player B decides to play strategy s_j^B against player A which plays the strategy s_i^A our total uncertainty about the pair (A, B) can be measured by an external “referee” through the joint entropy of the system. This is smaller or at least equal than the sum of the uncertainty about A and the uncertainty about B . The interaction and the correlation between A and B reduces the uncertainty due to the sharing of information. The uncertainty decreases while more systems interact jointly creating a new only system. If the same systems interact in separated groups the uncertainty about them is bigger. We can measure how much information A and B share and have an idea of how their strategies or states are correlated by its mutual or correlation entropy. If we know that B decides to play strategy s_j^B we can determinate the uncertainty about A through the conditional entropy.

Two external observers of the same game can measure the difference in their perceptions about certain strategy space of the same player A by its relative entropy. Each of them could define a relative frequency vector and the relative entropy over these two probability distributions is a measure of its closeness. We could also suppose that A could be in two possible states i.e. we know that A can play of two specific but different ways and each way has its probability distribution (“state”) that also is known. Suppose that this situation is repeated exactly N times or by N people. We can made certain “measure”, experiment or “trick” to determine which the state of the player is. The probability that these two states can be confused is given by the classical or the quantum Sanov’s theorem.

VII. MAXIMUM ENTROPY & THE COLLECTIVE WELFARE PRINCIPLE

We can maximize S by requiring that

$$\delta S = - \sum_i \delta \rho_{ii} (\ln \rho_{ii} + 1) = 0 \quad (21)$$

subject to the constrains $\delta Tr(\rho) = 0$ and $\delta \langle E \rangle = 0$. By using Lagrange multipliers

$$\sum_i \delta \rho_{ii} (\ln \rho_{ii} + \beta E_i + \gamma + 1) = 0 \quad (22)$$

and the normalization condition $Tr(\rho) = 1$ we find that

$$\rho_{ii} = \frac{e^{-\beta E_i}}{\sum_k e^{-\beta E_k}} \quad (23)$$

which is the condition that the density operator and its elements must satisfy to our system tends to maximize its entropy S . If we maximize S without the internal energy constrain $\delta \langle E \rangle = 0$ we obtain

$$\rho_{ii} = \frac{1}{N} \quad (24)$$

which is the $\beta \rightarrow 0$ limit (“high - temperature limit”) in equation (23) in where a canonical ensemble becomes a completely random ensemble in which all energy eigenstates are equally populated. In the opposite low - temperature limit $\beta \rightarrow \infty$ tell us that a canonical ensemble becomes a pure ensemble where only the ground state is populated. The parameter β is related to the “temperature” τ as follows

$$\beta = \frac{1}{\tau}. \quad (25)$$

By replacing ρ_{ii} obtained in the equation (23) in the von Neumann entropy we can rewrite it in function of the partition function $Z = \sum_k e^{-\beta E_k}$, β and $\langle E \rangle$ through the next equation

$$S = \ln Z + \beta \langle E \rangle. \quad (26)$$

It is easy to show that the next relationships for the energy of our system are satisfied

$$\langle E \rangle = - \frac{1}{Z} \frac{\partial Z}{\partial \beta} = - \frac{\partial \ln Z}{\partial \beta}, \quad (27)$$

$$\langle \Delta E^2 \rangle = - \frac{\partial \langle E \rangle}{\partial \beta} = - \frac{1}{\beta} \frac{\partial S}{\partial \beta}. \quad (28)$$

We can also analyze the variation of entropy with respect to the average energy of the system

$$\frac{\partial S}{\partial \langle E \rangle} = \frac{1}{\tau}, \quad (29)$$

$$\frac{\partial^2 S}{\partial \langle E \rangle^2} = - \frac{1}{\tau^2} \frac{\partial \tau}{\partial \langle E \rangle} \quad (30)$$

and with respect to the parameter β

$$\frac{\partial S}{\partial \beta} = - \beta \langle \Delta E^2 \rangle, \quad (31)$$

$$\frac{\partial^2 S}{\partial \beta^2} = \frac{\partial \langle E \rangle}{\partial \beta} + \beta \frac{\partial^2 \langle E \rangle}{\partial \beta^2}. \quad (32)$$

If our systems are analogous and thus exactly equivalents, our physical equilibrium should be also absolutely equivalent to our socioeconomic equilibrium. If in an

isolated system each of its accessible states do not have the same probability, the system is not in equilibrium. The system will vary and will evolve in time until it reaches the equilibrium state in where the probability of finding the system in each of the accessible states is the same. The system will find its more probable configuration in which the number of accessible states is maximum and equally probable. The whole system will vary and rearrange its state and the states of its ensembles with the purpose of maximize its entropy and reach its maximum entropy state. We could say that the purpose and maximum payoff of a quantum system is its maximum entropy state. The system and its members will vary and rearrange themselves to reach the best possible state for each of them which is also the best possible state for the whole system. This can be seen like a microscopical cooperation between quantum objects to improve its state with the purpose of reaching or maintaining the equilibrium of the system. All the members of our quantum system will play a game in which its maximum payoff is the equilibrium of the system. The members of the system act as a whole besides individuals like they obey a rule in where they prefer the welfare of the collective over the welfare of the individual. This equilibrium is represented in the maximum system entropy in where the system “resources” are fairly distributed over its members. A system where its members are in Nash Equilibrium (or ESS) is exactly equivalent to a system in a maximum entropy state [8, 9]. *“A system is stable only if it maximizes the welfare of the collective above the welfare of the individual. If it is maximized the welfare of the individual above the welfare of the collective the system gets unstable and eventually it collapses”* (Collective Welfare Principle [27, 28, 29, 31]).

There exist tacit rules inside a system. These rules do not need to be specified or clarified and search the system equilibrium under the collective welfare principle. The other “prohibitive” and “repressive” rules are imposed over the system when one or many of its members violate the collective welfare principle and search to maximize its individual welfare at the expense of the group. Then it is necessary to establish regulations over the system to try to reestablish the broken natural order.

Fundamentally, we could distinguish three states in every system: minimum entropy state, maximum entropy state, and when the system is tending to whatever of these two states. The natural trend of a physical system is to the maximum entropy state. The minimum entropy state is a characteristic of a “manipulated” system i.e. externally controlled or imposed. The “globalization” process has a behavior exactly equivalent to a physical system that is tending to a maximum entropy state. So, the analysis shown in this paper would predict the apparition of big common markets and strong common currencies as the characteristics of a system that is tending to that maximum entropy state. The system will find its equilibrium by decreasing its number of currencies and

markets until it gets a state characterized by only one common currency and only one common market.

VIII. THE WHY OF THE APPLICABILITY OF STATISTICAL PHYSICS TO ECONOMICS

Quantum mechanics could be a much more general theory that we had thought. It could encloses theories like EGT and evolutionary dynamics and we could explain through this theory biological and economical processes. From this point of view many of the equations, concepts and its properties defined quantically must be more general than its classical versions but they must remain inside the foundations of the new quantum version. So, our quantum equilibrium concept also must be more general than the one defined classically.

In our model we represent a population by a quantum system in which each subpopulation playing strategy s_i is represented by a pure ensemble in the state $|\Psi_k(t)\rangle$ and with probability p_k . The probability x_i of playing strategy s_i or the relative frequency of the individuals using strategy s_i in that population is represented by the probability ρ_{ii} of finding each pure ensemble in the state $|i\rangle$. Through these quantization relationships the replicator dynamics (in matrix commutative form) takes the form of the equation of evolution of mixed states. The quantum analogue of the relative frequencies matrix is the density operator. The relationships between these two systems described by these two matrixes and their evolution equations would let us analyze the entropy of our system through the well known von Neumann entropy in the quantum case and by the Shannon entropy in the classical case. The properties that these entropies enjoy would let us analyze a “game” from a different point of view through information and a maximum or minimum entropy criterion.

Every game can be described by a density operator, the von Neumann entropy and the quantum replicator dynamics. The density operator is maybe the most important tool in quantum mechanics. From the density operator we can construct and obtain all the statistical information about our system. Also we can develop the system in function of its information and analyze it through information theories under a criterion of maximum or minimum entropy. There exists a strong relationship between game theories, statistical mechanics and information theory. The bonds between these theories are the density operator and entropy.

It is important to remember that we are dealing with very general and unspecific terms, definitions, and concepts like state, game and system. Due to this, the theories that have been developed around these terms like quantum mechanics, statistical physics, information theory and game theory enjoy of this generality quality and

could be applicable to model any system depending on what we want to mean for game, state, or system. Objectively these words can be and represent anything. Once we have defined what system is in our model, we could try to understand what kind of “game” is developing between its members and how they accommodate their “states” in order to get their objectives. This would let us visualize what temperature, energy and entropy would represent in our specific system through the relationships, properties and laws that were defined before when we described a physical system.

Entropy can be defined over any random variable and can be maximized subject to different constraints. In each case the result is the condition the system must follow to maximize its entropy. Generally, this condition is a probability distribution function. For the case analyzed in this paper, this distribution function depends on certain parameter “ β ” which is related inversely with the system “temperature”. Depending on what the variable over which we want to determine its grade of order or disorder is we can resolve if the best for the system is a state of maximum or minimum entropy. If we would measure the order or disorder of our system over a resources distribution variable the best state for that system is those in where its resources are fairly distributed over its members which would represent a state of maximum entropy. By the other hand, if we define an entropy over an acceptance relative frequency of a presidential candidate in a democratic process the best would represent a minimum entropy state i.e. the acceptance of a candidate by the vast majority of the population.

IX. CONCLUSIONS

By analyzing the relationships between the systems described through quantum mechanics and game theory we showed that both systems are analogous and thus exactly equivalents. The extensions of quantum mechanics to statistical physics and information theory let us use some of their concepts and definitions for the best understanding of the behavior of economics and biology. Quantum mechanics could be a much more general the-

ory that we had thought. It could enclose theories like EGT and evolutionary dynamics.

Due to the generality of the terms state, game and system, quantum mechanics, statistical physics, information theory and game theory enjoy also of this generality quality and could be applicable to model any system depending on what we want to mean for game, state, or system. Once we have defined what the term system is in our model, we could try to understand what kind of “game” is developing between its members and how they accommodate its “states” in order to get their objectives.

Entropy can be defined over any random variable and can be maximized subject to different constraints. Depending on what the variable over which we want to determine its grade of order or disorder is we can resolve if the best for the system is its state of maximum or minimum entropy. A system can be internally or externally controlled with the purpose of guide it to a state of maximum or minimum entropy depending of the ambitions of the members that compose it or the “people” who control it. Every “game” could be described by a density operator with its entropy equal to von Neumann’s and its evolution equation given by the quantum replicator dynamics which is the von Neumann equation.

The quantum analogue of the replicator dynamics is the von Neumann equation. A system in where all its members are in Nash equilibrium is equivalent to a system in a maximum entropy state. Nature is a game in where its players compete for a common welfare and the equilibrium of the system that they are members. They act as a whole besides individuals like they obey a rule in where they prefer to work for the welfare of the collective besides the individual welfare. This equilibrium is represented in the maximum system entropy in where the system “resources” are fairly distributed over its members. A system is stable only if it maximizes the welfare of the collective above the welfare of the individual. If it is maximized the welfare of the individual above the welfare of the collective the system gets unstable and eventually it collapses.

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- [1] M. Ausloos, N. Vandewalle, Ph. Boveroux, A. Minguet, K. Ivanova, *Physica A* **247**, 229-240 (1999).
 - [2] J.-P. Bouchaud, *An introduction to statistical finance*, *Physica A* **313**, 238-251 (2002).
 - [3] H. Eugene Stanley, *Exotic statistical physics: Applications to biology, medicine, and economics*, *Physica A* **285**, 1-17 (2000).
 - [4] L.Ya. Kobelev, O.L. Kobeleva, Ya.L. Kobelev, *Is it Possible to Describe Economical Phenomena by Methods of Statistical Physics of Open Systems?*, physics/0005010.
 - [5] Masanao Aoki, *New approaches to Macroeconomics Modeling* (Cambridge University Press, Cambridge, 1996).
 - [6] I. Antoniou, V.V. Ivanov, Yu.L. Korolev, A.V. Kryanev, V.V. Matokhin, Z. Suchanecki, *Physica A* **304**, 525-534 (2002).
 - [7] A. Drăgulescu and V. M. Yakovenko, *Statistical mechanics of money*, *Eur. Phys. J. B* **17**, 723-729 (2000).
 - [8] A. Darooneh, *Entropy* **8**[1], 18-24 (2006).
 - [9] F. Topsøe, *Information Theoretical Optimization Techniques*, *Kybernetika* **15**, 8-27 (1979). F. Topsøe, *Game theoretical equilibrium, maximum entropy and minimum information discrimination*, in *Maximum Entropy and*

- Bayesian Methods* (A. Mohammad-Djafari and G. Demoments (eds.), pp. 15-23, Kluwer, Dordrecht, 1993).
- [10] J. von Neumann and O. Morgenstern, *The Theory of Games and Economic Behavior* (Princeton University Press, Princeton, 1947).
 - [11] R. B. Myerson, *Game Theory: An Analysis of Conflict* (MIT Press, Cambridge, 1991).
 - [12] M. A. Nowak and K. Sigmund, *Nature* **398**, 367 (1999).
 - [13] J. M. Smith, *Evolution and The Theory of Games* (Cambridge University Press, Cambridge, UK, 1982).
 - [14] J. Hofbauer and K. Sigmund, *Evolutionary Games and Replicator Dynamics* (Cambridge University Press, Cambridge, UK, 1998).
 - [15] J. Weibull, *Evolutionary Game Theory* (MIT Press, Cambridge, MA, 1995).
 - [16] R. A. Fisher, *The Genetic Theory of Natural Selection* (Oxford, Clarendon Press, 1930).
 - [17] P. Hammerstein and R. Selten, *Game Theory and Evolutionary Biology* (Handbook of Game Theory. Vol 2. Elsevier B.V., 1994).
 - [18] P. D. Taylor and L. B. Jonker, *Evolutionary stable strategies and game dynamics*, *Mathematical Biosciences* **40**, 145-156 (1978).
 - [19] R. Cressman, *The Stability Concept of Evolutionary Game Theory: A Dynamic Approach* (Springer-Verlag, New York, 1992).
 - [20] J. M. Smith and G. R. Price, *The logic of animal conflict*, *Nature* **246**, 15 (1973).
 - [21] D. A. Meyer, *Phys. Rev. Lett.* **82**, 1052-1055 (1999).
 - [22] J. Eisert, M. Wilkens and M. Lewenstein, *Phys. Rev. Lett.* **83**, 3077 (1999).
 - [23] L. Marinatto and T. Weber, *Phys. Lett. A* **272**, 291 (2000).
 - [24] A. P. Flitney and D. Abbott, *Proc. R. Soc. (London) A* **459**, 2463-74 (2003).
 - [25] E. W. Piotrowski and J. Sladkowski, *Int. J. Theor. Phys.* **42**, 1089 (2003).
 - [26] Azhar Iqbal, PhD thesis, Quaid-i-Azam University, 2004, quant-ph/0503176.
 - [27] E. Guevara H., *Quantum Replicator Dynamics*, *Physica A* **369/2**, 393-407 (2006).
 - [28] E. Guevara H., *The Why of the applicability of Statistical Physics to Economics*, physics/0609088.
 - [29] E. Guevara H., *Quantum Econophysics*, in Proceedings of Quantum Interaction 2007, AAAI Spring Symposia Series, Stanford University, Palo Alto, published by the American Association of Artificial Intelligence, AAAI PRESS Technical Report **SS-07-08**, 158-165 (2007).
 - [30] E. Guevara H., *Introduction to the study of entropy in Quantum Games*, quant-ph/0604170.
 - [31] E. Guevara H., *Quantum Games Entropy*, *Physica A* (to be published), quant-ph/0606045.
 - [32] J. von Neumann, *Thermodynamik quantenmechanischer Gesamtheiten*, *Gött. Nach.* **1** 273-291(1927).
 - [33] J. von Neumann, *Mathematische Grundlagen der Quantenmechanik* (Springer, Berlin, 1932).
 - [34] B. Schumacher and M. D. Westmoreland, *Phys. Rev. A* **56**, 131 (1997).
 - [35] A. S. Holevo, *IEEE Trans. Inf. Theory* **44**, 269 (1998).
 - [36] B. Schumacher, *Phys. Rev. A* **51**, 2738 (1995).
 - [37] R. Jozsa and B. Schumacher, *J. Mod. Opt.* **41**, 2343 (1994).
 - [38] C. H. Bennett and D. P. DiVincenzo, *Nature* **377**, 389 (1995).
 - [39] D. P. DiVincenzo, *Science* **270**, 255 (1995).
 - [40] C. H. Bennett and S. J. Wiesner, *Phys. Rev. Lett.* **69**, 2881 (1992).
 - [41] C. H. Bennett et al., *Phys. Rev. Lett.* **70**, 1895 (1993).
 - [42] A. Ekert, *Nature* **358**, 14 (1992).
 - [43] C. H. Bennett, G. Brassard, and N. D. Mermin, *Phys. Rev. Lett.* **68**, 557 (1992).